International Journal of Theoretical Physics, Vol. 14, No. 3 (1975), pp. 207–210

Higher Energy Levels of the Harmonic Oscillator and Stochastic Electrodynamics

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Received: 30 *December* 1974

Abstract

Higher energy levels of the harmonic oscillator and the "radial quantification" formula for the hydrogen atom are obtained within the framework of stochastic electrodynamics. In two remarks, intricacies of quantum mechanics are highlighted.

The simplicity of its axiomatics and the "classical" nature of stochastic electrodynamics (SED) have prompted the author to apply SED to various physical problems. Thus, some new results and numerous quantumlike results were obtained (Surdin, 1971a, 1971b; 1973; 1974a, 1974b). The present Note is part of the program of the investigation into the possibilities of SED.

Using the concept of the universal fluctuating electromagnetic field at the absolute zero of temperature, the zero-point field, one may obtain its energy spectrum (Braffort & Tzara, 1954); in the one-dimensional case one has

$$\epsilon(\omega) = K\omega^3/3\pi c^3 \tag{1}$$

where K is a constant having the dimension of action.

With this unique quantitative information about the zero-point field it was possible to show (Braffort et al., 1965) that the average energy of a one-dimensional harmonic oscillator is

$$\langle E(\omega, T = 0)\rangle = \frac{K\omega_0}{2}\left(1 - \frac{1}{2\pi}\frac{\omega_0}{\omega_s}\log\frac{\omega_0}{\omega_s}\right) \tag{2}$$

where ω_0 is the resonant frequency of the oscillator and $\omega_s = 3mc^3/2e^2$; the second term in the right-hand side of Eq. (2) corresponds to the Lamb shift.

It was also possible to show that if one considers an ensemble of harmonic oscillators in thermal equilibrium with a thermostat at temperature T, then the average energy of an oscillator at temperature T is given by (Surdin et al., 1966)

$$\langle E(\omega, T)\rangle = \frac{K\omega}{2}\left(1 + \frac{2}{e^{K\omega/kT} - 1}\right) \tag{3}$$

which is Planck's law for black-body radiation.

It was deemed interesting to examine the possibility of obtaining the expression giving the higher energy levels of the harmonic oscillator when using the same kind of arguments.

The proper approach to this problem appears to be the following: One considers the harmonic oscillator in the presence of the zero-point field and a monochromatic electromagnetic field of frequency ν. Then one computes the expression, as a function of ν and ω_0, of the energy absorbed from the monochromatic electromagnetic field by the harmonic oscillator.

In the present Note a somewhat different, not exactly equivalent, approach is used: One considers, as in the case of the black-body radiation, an ensemble of harmonic oscillators in thermal equilibrium with a thermostat at temperature T and one derives the expression of E_n, for higher energy levels, of the individual harmonic oscillator.

If one could derive an alternate relation for Eq. (3), giving $\langle E(\omega, T)\rangle$, a comparison between these two relations may yield the expression giving E_n. Obviously, such a relation may be obtained from the Gibbs distribution, viz.,

$$\langle E(\omega, T)\rangle = \frac{\sum_n E_n e^{-E_n/kT}}{\sum_n e^{-E_n/kT}} \tag{4}$$

Let $K\omega/kT = x$, then it appears that the expansion of $1/(e^x - 1)$ in powers of e^{-x}, equivalent to the right-hand side of Eq. (4), has a well-known unique solution (Tolman, 1938),

$$\frac{1}{e^x - 1} = \frac{e^{-x} + 2e^{-2x} + 3e^{-3x} + 4e^{-4x} + \cdots}{1 + e^{-x} + e^{-2x} + e^{-3x} + e^{-4x} + \cdots} \tag{5}$$

Taking into account the first term of the right-hand side of Eq. (3) and rearranging, one obtains from Eqs. (3) and (5)

$$\langle E(\omega, T)\rangle = \frac{\sum_{n=0}^{\infty} (n + \frac{1}{2})K\omega e^{-(n+\frac{1}{2})K\omega/kT}}{\sum_{n=0}^{\infty} e^{-(n+\frac{1}{2})K\omega/kT}}, \qquad n = 0, 1, 2, 3, \ldots \tag{6}$$

Comparing Eqs. (4) and (6), we obtain

$$E_n = (n + \tfrac{1}{2})K\omega, \qquad n = 0, 1, 2, 3, \ldots \tag{7}$$

The above result may be used to derive the expression of E_n for the excited states of the hydrogen atom.

In classical mechanics the orbit of a body moving in a central force field is planar. Consider in the plane of the orbit two orthogonal axes $0x$, $0y$. The motion of an electron in the Coulomb field potential $-e^2/r$, in the presence of the zero-point field, can be considered as the resultant of the motions of two one-dimensional harmonic oscillators, having the same resonant frequency, oscillating independently along the two orthogonal axes.

Let $\omega_0{}^2 = e^2/mr_n{}^3$ be the resonant frequency; r_n is the "average radius" of the orbit. It was shown (Surdin, 1973) that, owing to the fact that the resonance curve of the oscillator is very narrow, the average kinetic and the average potential energies of the oscillator are the same, so that

$$\begin{aligned} (n_1 + \tfrac{1}{2})K\omega_0 &= m\omega_0{}^2 \langle x^2 \rangle \\ (n_2 + \tfrac{1}{2})K\omega_0 &= m\omega_0{}^2 \langle y^2 \rangle \end{aligned} \tag{8}$$

Hence

$$r_n{}^2 = (n_1 + n_2 + 1)K/m\omega_0 \tag{9}$$

Since the two harmonic oscillators are independent, one may write

$$n_1 + n_2 = n, \qquad n = 0, 1, 2, 3, \ldots \tag{10}$$

hence

$$r_n{}^2 = (n + 1)K/m\omega_0 \tag{10}$$

Replacing in Eq. (10) ω_0, one obtains

$$r_n = (n + 1)^2K^2/e^2m, \qquad n = 0, 1, 2, 3, \ldots \tag{11}$$

which is the radial quantification expression in quantum mechanics (QM).*

The preceding suggests two interesting remarks:

(1) In QM Eq. (3) may be obtained from statistical considerations. The derivation given above of Eq. (7) from Eq. (3) is valid in QM. In other words, Eq. (7) may be obtained in QM from ensemble considerations. However, Eq. (7) is also obtained in QM as a solution of the Schrödinger equation, which involves a single harmonic oscillator. It appears that here one comes up against the well-known difficulties in the interpretation of QM. In SED the difficulty does not arise. Equations (3) and (7) are obtained by considering ensemble averages. The modified Schrödinger equation was obtained by considering time averages (Surdin, 1971a). Since the ergodic hypothesis is verified in SED, no contradiction may arise.

(2) Equation (7), as derived in QM from Eq. (3), involves an ensemble of harmonic oscillators in thermal equilibrium with a thermostat at temperature T. Equation (7) is also a solution of Schrödinger's equation, where no mention is made of temperature T. In SED the modified Schrödinger equation is ob-

* The same results were obtained by a somewhat different approach in an unpublished note by P. Braffort, M. Surdin, and A. Taroni in 1966.

tained for a harmonic oscillator in the presence of the zero-point field. The left-hand side of this equation [Schrödinger's part, see Eq. (3.13) of Surdin (1971a)], remains the same if the fluctuating electromagnetic field is at temperature T, provided the process remains Markoffian. The only term which is modified for $T \neq 0$ is the S term, in the right-hand side of this equation. This remark may explain why Schrödinger's equation is independent of T in *QM.*

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